

## Electromagnetic Neutrino Properties and Neutrino Oscillations in Electromagnetic Fields

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*Introduction.* The problem of neutrino from the time when this particle was theoretically recognized (Pauli, 1930; Fermi, 1932-1934) as an important element of the theory of weak interaction, still is closely connected with the fundamentals of particle physics. That is why neutrino properties are especially interesting. However, inspite of great efforts, there is a set of open basic questions about neutrinos (nonzero mass; number of flavours; mixing between the neutrino families; behavior under particle-antiparticle conjugation; electromagnetic properties; the role in astrophysics etc.). For sure the answer to any of these questions will substantially stimulate progress in understanding of laws of micro and macro worlds.

Electromagnetic moments are among the most important properties of a particle. Since neutrinos are neutral particles and there is no coupling to the photon in the lagrangian, electromagnetic moments arise entirely from vacuum polarization loop effects. For neutrino as a spin- $\frac{1}{2}$  particle the matrix element of electromagnetic current contains four independent form factors<sup>1</sup>:

$$\begin{aligned} \langle \nu(p') | J_\mu^{em} | \nu(p) \rangle = & \vec{u}_f [f_Q(q^2)\gamma_\mu + f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5 \\ & + f_M(q^2)\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5] u_i, \quad q = p_f - p_i, \end{aligned} \quad (1)$$

where  $f_Q$ ,  $f_A$ ,  $f_M$  and  $f_E$  are the charge, axial charge, magnetic and electric dipole moments, respectively. For Dirac neutrinos the assumption of CP invariance combined with the hermicity of the electromagnetic current  $J_\mu^{em}$  implies that one of two possible static moments,  $f_M$  and  $f_E$ , vanishes:  $f_E = 0$ . For neutral Majorana neutrinos from a general assumption of CPT invariance it follows that both magnetic and electric dipole moments vanish. However, in the case of the off-diagonal electromagnetic vertex the neutrino transition magnetic moment is not, in general, zero.

Thus, one can conclude that the neutrino magnetic (transition) moment is the most important among the form factors. It should be mentioned here, that the discussion on the third non trivial electromagnetic characteristic of neutrino,  $f_A$ , can be found in<sup>2</sup>. The magnetic moment arises from the operator  $\sigma_{\mu\nu}q^\nu$  and since  $\vec{\Psi}'\sigma_{\mu\nu}\Psi = \vec{\Psi}'_L\sigma_{\mu\nu}\Psi_R + \vec{\Psi}'_R\sigma_{\mu\nu}\Psi_L$ , it follows that a chirality change can appear when there are both left and right-handed particle states.

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In the standard model due to the absence of the right-handed charged currents a massless Dirac neutrino cannot have a magnetic moment. In the standard model supplied with  $SU(2)$ -singlet right-handed neutrino  $\nu_R$  the one-loop radiative correction generates nonvanishing magnetic moment, proportional to  $m_\nu$ <sup>3,4</sup>:

$$\mu_\nu = \frac{3eG_F}{8\sqrt{2}\pi^2} m_\nu = 3 \cdot 10^{-19} \left(\frac{m_\nu}{1\text{eV}}\right) \mu_0, \quad \mu_0 = \frac{e}{2m_e}. \quad (2)$$

There are plenty of models<sup>5</sup> which predict much larger magnetic moments for neutrinos. In some of these models<sup>6</sup> the neutrino magnetic moment can be lifted up to the range  $\mu \sim 10^{-10} \div 10^{-12} \mu_0$  that could be of practical interest for processes in the vicinity of the Sun, neutron star and supernova.

Experimental upper bounds on the neutrino magnetic moments are<sup>7</sup>:  $\mu_{\nu_e} \leq 1.8 \cdot 10^{-10} \mu_0$ ,  $\mu_{\nu_\mu} \leq 7.4 \cdot 10^{-10} \mu_0$ ,  $\mu_{\nu_\tau} \leq 5.4 \cdot 10^{-7} \mu_0$ . One could also obtain more stringent constraints from astrophysical arguments like neutrino emission from red giant stars<sup>8</sup> or from supernovae and neutron stars<sup>9,10,11</sup>.

*Neutrino magnetic moment in electromagnetic field.* It is non-trivial result shown for the first time in<sup>12</sup>, that the presence of medium and external magnetic field change electromagnetic properties of neutrino. Thus the massless neutrino in the standard model due to the weak interaction with particles of the medium can get an effective magnetic moment. In an analogy with the case of charged lepton magnetic moment (see, e.g.,<sup>13</sup>) the non vanishing intrinsic neutrino magnetic moment also depends on the strength of external electromagnetic field and the energy of the neutrino. The most systematic way in investigation of the external field and energy dependence of neutrino magnetic moment is based on consideration of the neutrino mass operator  $\hat{M}(x', x)$  in electromagnetic field that accounts for the field radiative corrections to the neutrino motion and defines the righthand side of the Schwinger equation

$$(i \not{p} - m_\nu) \Psi_\nu(x) = \int \hat{M}(x', x, F) \Psi_\nu(x') dx'. \quad (3)$$

We consider the lowest-order standard model contribution (i.e. the contribution of the virtual loop process  $\nu \rightarrow e + W \rightarrow \nu$ ) to the Dirac neutrino mass operator in external crossed electromagnetic field ( $\vec{E} \perp \vec{B}$ ,  $E = B$ ). The details of calculations of a lepton magnetic moment in the external magnetic field can be found in<sup>14</sup>. Finally, for the neutrino magnetic moment in the external field we find

$$\begin{aligned} \frac{\mu_\nu^W(x)}{\mu_0} &= \frac{g^2}{2^5 \pi^2} \frac{m_\nu}{m_e} \int_0^\infty \frac{du}{(1+u)^{8/3}} \frac{u^{1/3}}{\chi^{2/3}} \left[ \frac{2(2+u)}{u} - \frac{\eta}{\lambda} + \frac{(2u+1)}{\lambda} \right] \Upsilon(z), \quad \eta = \frac{m_e^2}{m_\nu^2}, \\ \lambda &= \frac{m_W^2}{m_e^2}, \quad z = \left(\frac{u}{\chi}\right)^{2/3} \left(\frac{1+u}{u}\right)^{1/3} \left(1 + \frac{\lambda}{u} - \frac{\eta}{1+u}\right), \quad \Upsilon(z) = \int_0^\infty \sin\left(zx + \frac{x^3}{3}\right) dx, \end{aligned} \quad (4)$$

where  $g$  is the coupling constant of the standard model,  $\chi = [-(eF^{\mu\nu}p_\nu)^2]^{1/2}$ .  $m_e^{-3}$  is the field dynamical parameter,  $p_\nu$  is the neutrino momentum. The received expression for  $\mu_\nu^W(\chi)$  exactly accounts for dependence on the field parameter  $\chi$  and also on masses  $m_\nu$ ,  $m_e$ , and  $m_W$ . If in (4) one neglect the terms  $\sim \frac{m_e^2}{m_W^2}$ ,  $\frac{m_\nu^2}{m_W^2}$ , then the result of <sup>15</sup> is achieved. It is possible to extract the vacuum field and energy independent part of  $\mu_\nu^W$ :

$$\begin{aligned}\mu_\nu^W(0) &= \frac{g^2 e}{(8\pi)^2} \frac{m_\nu}{m_e^2} F(\lambda, \eta), \\ F(\lambda, \eta) &= \frac{1}{\eta} \left\{ 2 + \frac{\eta}{\lambda} + \frac{1}{\lambda} + \left[ \frac{1}{\eta} \left( \frac{1}{2\lambda} - \lambda + \frac{1}{2} \right) - \frac{1}{2\lambda} + \frac{3}{2} \right] \ln \lambda \right. \\ &\quad \left. + \left[ \eta \left( \frac{3}{2} + \frac{1}{2\lambda} \right) + \frac{1}{2} - \frac{1}{\lambda} - \frac{5}{2} \lambda + \frac{1}{\eta} \left( \lambda^2 - \frac{3}{2} \lambda + \frac{1}{2\lambda} \right) \right] I(\lambda, \eta) \right\}, \\ I(\lambda, \eta) &= -\frac{1}{\sqrt{\Delta}} \ln \frac{\epsilon - \sqrt{\Delta}}{\epsilon + \sqrt{\Delta}}, \Delta = \epsilon^2 - 4\eta, \epsilon = 1 + \lambda - \eta.\end{aligned}\tag{5}$$

In the case of small  $m_\nu$ ,  $\eta \ll 1$ , the function  $F$  is  $F(\lambda, 0) = \frac{3\lambda-1}{(\lambda-1)^2} + \frac{\lambda}{(\lambda-1)^3} \ln \lambda$ . In the limit  $\lambda \gg 1$  we get  $F(\lambda, 0) \approx 3/\lambda$ , that together with (5) reproduce the result of eq.(2). In analogous way we have also considered <sup>16</sup> the other possible contributions to the neutrino magnetic moment, that are predicted within extensions of the standard model.

*Neutrino conversion in electromagnetic field.* Consider a system,  $\nu = (\nu_R, \nu_L)$ , of two neutrino chiral components in a magnetic field  $\vec{B}$ . The evolution of  $\nu$  can be written as a Schrödinger like equation ( see <sup>17</sup> and also <sup>18</sup>)

$$i \frac{\partial}{\partial t} \nu = H \nu, \quad H = (\vec{\sigma} \vec{n}) \left( \frac{\Delta m^2 A}{4E} - \frac{V}{2} \right) - \mu \vec{\sigma} \left( \vec{B} - \vec{n}(\vec{B} \vec{n}) \right), \tag{6}$$

where  $\vec{n}$  is the unit vector in the direction of neutrino speed  $\vec{u}$ ,  $\vec{\sigma}$  is the Pauli matrixes,  $V$  is difference of neutrino effective potentials in matter,  $A$  is a function of the neutrino vacuum mixing angle  $\theta$  which is determined by the considered type of neutrino conversion process (for example,  $A = \frac{1}{2}(\cos \theta - 1)$  for  $\nu_{eL} \leftrightarrow \nu_{eR}$ ). In the Hamiltonian (6) the term that is proportional to the unit matrix is omitted. Equation (6) can be received in the frame of generalized standard models of electroweak interactions. However, it can be derived from some general arguments within quasiclassical approach. It is well known that in the classical approximation the evolution of particle spin can be determined by the Bargmann-Michel-Telegdi equation <sup>19</sup>. On the basis the BMT equation one can get <sup>20</sup> the following equation for the spin vector  $S^\mu$  of the neutral particle moving with constant speed,  $u_\mu = const$ , in electromagnetic field  $F_{\mu\nu}$ :

$$\frac{dS^\mu}{d\tau} = 2\mu \{ F^{\mu\nu} S_\nu - u^\mu (u_\nu F^{\nu\lambda} S_\lambda) \} + 2\epsilon \{ \tilde{F}^{\mu\nu} S_\nu - u^\mu (u_\nu \tilde{F}^{\nu\lambda} S_\lambda) \}. \tag{7}$$

Here  $\tilde{F}_{\mu\nu}$  is the dual electromagnetic field tensor, and we account for possibility of non-vanishing electric dipole moment,  $\epsilon$ . Note that the second term in eq.(7)

violate  $T$  invariance. Let us generalize eq.(7) for the case of models with  $CP$  invariance and  $P$  nonconservation. A  $P$ -noninvariant theory implies existence of a preferred direction that can be determined by the constant vector  $\vec{k}$ . The Lorentz invariant generalization of the eq.(7) is given by the substitution  $F_{\mu\nu} \rightarrow F_{\mu\nu} + G_{\mu\nu}$ , where  $G_{\mu\nu}$  is an anti symmetric tensor organized as follows:  $G_{\mu\nu} = (\xi\vec{k}, \rho\vec{k})$ . If one demands that the generalized equation should be linear over field  $F_{\mu\nu}$  then the only possibility is to identify vector  $\vec{k}$  with unit vector  $\vec{n}$ :  $\vec{k} = \vec{n} = \frac{\vec{u}}{u}$ . The quantities  $\xi$  and  $\rho$  are pseudo scalar and scalar, respectively, and they do not depend on  $F_{\mu\nu}$ . In the rest frame of particle for the model with  $T$  invariance we can get

$$\frac{d\vec{S}}{dt} = 2[\vec{S} \times \vec{R}], \quad \vec{R} = \frac{\mu}{u_0}(\vec{B}_0 + \rho\vec{n}) \quad (8)$$

where  $\vec{B}_0 = u_0\vec{B} + [\vec{E} \times \vec{u}] - \frac{\vec{u}(\vec{u}\vec{B})}{1+u_0}$  is the magnetic field in the rest frame. It follows that even in the absence of electromagnetic field spin precession exist if the spin  $\vec{S}$  is not exactly parallel with velocity  $\vec{u}$ . If for description of the neutrino spin states we use the spin tensor  $S = (\vec{\sigma}\vec{S})$  the evolution of which is determined by  $S(t) = US(t_0)U^+$ , then we can obtain <sup>21</sup>

$$\frac{dU}{dt} = i(\vec{\sigma}\vec{R})U. \quad (9)$$

This equation will coincide with one for the evolution operator that determines solutions of the Schrödinger eq.(6) if we take  $\rho = \frac{u_0}{\mu}(\frac{V}{2} - \frac{\Delta m^2 A}{4E})$  and the transversal magnetic field  $\vec{B}_\perp = \vec{B} - \vec{n}(\vec{B}\vec{n})$  is substituted by  $\frac{\vec{B}_0}{u_0}$ .

Now we consider the neutrino spin precession in a field of electromagnetic wave with frequency  $\omega$ . We will denote by  $\vec{e}_3$  the axis that is parallel with  $\vec{u}$  and by  $\phi$  the angle between  $\vec{e}_3$  and the wave vector of the wave. The magnetic field in the rest frame is given by

$$\vec{B}_0 = u_0[(\cos\phi - \beta)B_1\vec{e}_1 + (1 - \beta\cos\phi)B_2\vec{e}_2 - \frac{\sin\phi}{u_0}B_1\vec{e}_3], \quad (10)$$

where  $\beta = \frac{u}{u_0}$ ,  $\vec{e}_{1,2,3}$  are the unit orthogonal vectors. For the wave of circular polarization  $B_1 = B\cos\psi$ ,  $B_2 = B\sin\psi$ , and the phase  $\psi = g\omega t(1 - \frac{\beta}{\beta_0}\cos\phi)$  depends on the wave speed  $\beta_0$  ( $\beta_0 \leq 1$ ,  $g = \pm 1$ ). Expanding over small parameter  $1/u_0 \ll 1$  and neglecting terms which are proportional to  $\frac{\mu B}{u_0}$ , we get

$$\vec{R} = \left(\frac{V}{2} - \frac{\Delta m^2 A}{4E}\right)\vec{e}_3 - \mu B(1 - \beta\cos\phi)(\vec{e}_1\cos\psi - \vec{e}_2\sin\psi) + O(\frac{\mu B}{u_0}). \quad (11)$$

In this case the solution of eq.(9) can be written in the form

$$U = U_{\vec{e}_3}(\psi - \psi_0)U_{\vec{l}}(\chi - \chi_0), \quad U_{\vec{l}}(\chi) = \exp(i\frac{(\vec{\sigma}\vec{l})}{l}\frac{\chi}{2}). \quad (12)$$

The evolution operator is a combination of the operator which describes rotation on angle  $\chi - \chi_0 = 2l(t - t_0)$  around the axis  $\vec{l}$ , and the rotation operator on the angle  $\psi - \psi_0$  around the axis  $\vec{e}_3$ . For the vector  $\vec{l}$  we get

$$\vec{l} = \left( \frac{V}{2} - \frac{\Delta m^2 A}{4E} - \frac{\dot{\psi}}{2} \right) \vec{e}_3 - \mu B (1 - \beta \cos \phi) (\vec{e}_1 \cos \psi_0 - \vec{e}_2 \sin \psi_0). \quad (13)$$

The conversion probability among the two neutrino states,  $\nu_L$  and  $\nu_R$ , could become sufficient when the vector  $\vec{l}$  is orthogonal or nearly orthogonal to the axis  $\vec{e}_3$ . This will happen when the condition

$$\left| \frac{V}{2} - \frac{\Delta m^2 A}{4E} - \frac{\dot{\psi}}{2} \right| \ll \mu B (1 - \beta \cos \phi) \quad (14)$$

is valid. This inequality binds together the properties of neutrino ( $\mu$ ,  $\Delta m^2$ ,  $E$ ,  $\theta$ ) and medium ( $V$ ), as well as the direction of propagation and the other characteristics of the electromagnetic wave. Condition (14) predicts the new type of neutrino resonances  $\nu_L \leftrightarrow \nu_R$  in the electromagnetic field.

In our previous studies<sup>10,17</sup> we discussed neutrino conversion and oscillations among the two neutrino species induced by strong twisting magnetic field,  $\vec{B} = \vec{B}_\perp e^{i\psi(t)}$ . We introduced the critical magnetic field  $\tilde{B}_{cr}(\Delta m^2, \theta, n_{eff}, E, \dot{\psi}(t))$  as a function of the neutrino properties and the angle  $\psi$  specifying the variation of the magnetic field in the plane transverse to the neutrino motion, that determines the range of fields ( $B \geq B_{cr}$ ) for which the magnetic field induced neutrino conversion become significant. For the critical field we got

$$\tilde{B}_{cr} = \left| \frac{1}{2\mu} \left( \frac{\Delta m^2 A}{2E} - \sqrt{2} G_F n_{eff} + \dot{\psi} \right) \right|. \quad (15)$$

Remarkably, this result follows from (14) if  $\cos \phi = 0$ . It was also pointed out<sup>10,17</sup> that effects of the magnetic field induced neutrino conversion become important if the following two conditions are satisfied: 1) the magnetic field exceeds the critical value  $B_{cr}$  ( $B \geq B_{cr}$ ) and 2) the length  $x$  of the neutrinos path in the medium must be greater than the effective oscillation length  $L_{eff}$ ,  $x \geq \frac{L_{eff}}{2}$ . We used this criterion to get constraints on  $\mu$ . In particular, to avoid the loss of a substantial amount (25%) of energy that could escape during the supernova explosion<sup>22</sup> together with the sterile neutrinos  $\nu_{eR}$ , we have to constrain the magnetic moment on the level of  $\mu \leq 10^{-11} \mu_0$ . More stringent constraint  $\mu \leq 10^{-15} \mu_0$  is also obtained<sup>11</sup> from consideration of the magnetized neutron star cooling.

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